Adama Science and Technology University

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Fundamentals of Electrical Engineering (EPCE210)

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CHAPTER 8:

MAGNETICALLY COUPLED CIRCUIT

SUB - TOPICS

- SELF AND MUTUAL INDUCTANCE.
- COUPLING COEFFICIENT (K)
- DOT DETERMINATION

OBJECTIVES

- To understand the basic concept of self inductance and mutual inductance.
- To understand the concept of coupling coefficient and dot determination in circuit analysis.

SELF AND MUTUAL INDUCTANCE

- When two loops with or without contacts between them affect each other through the magnetic field generated by one of them, it called *magnetically coupled*.
- Example: transformer
  - ➢ An electrical device designed on the basis of the concept of magnetic coupling.
  - ➢ Used magnetically coupled coils to transfer energy from one circuit to another.

a) Self Inductance

- It called *self inductance* because it relates the voltage induced in a coil by a time varying current in the same coil.
- Consider a single inductor with N number of turns when current, *i* flows through the coil, a magnetic flux, Φ is produces around it.

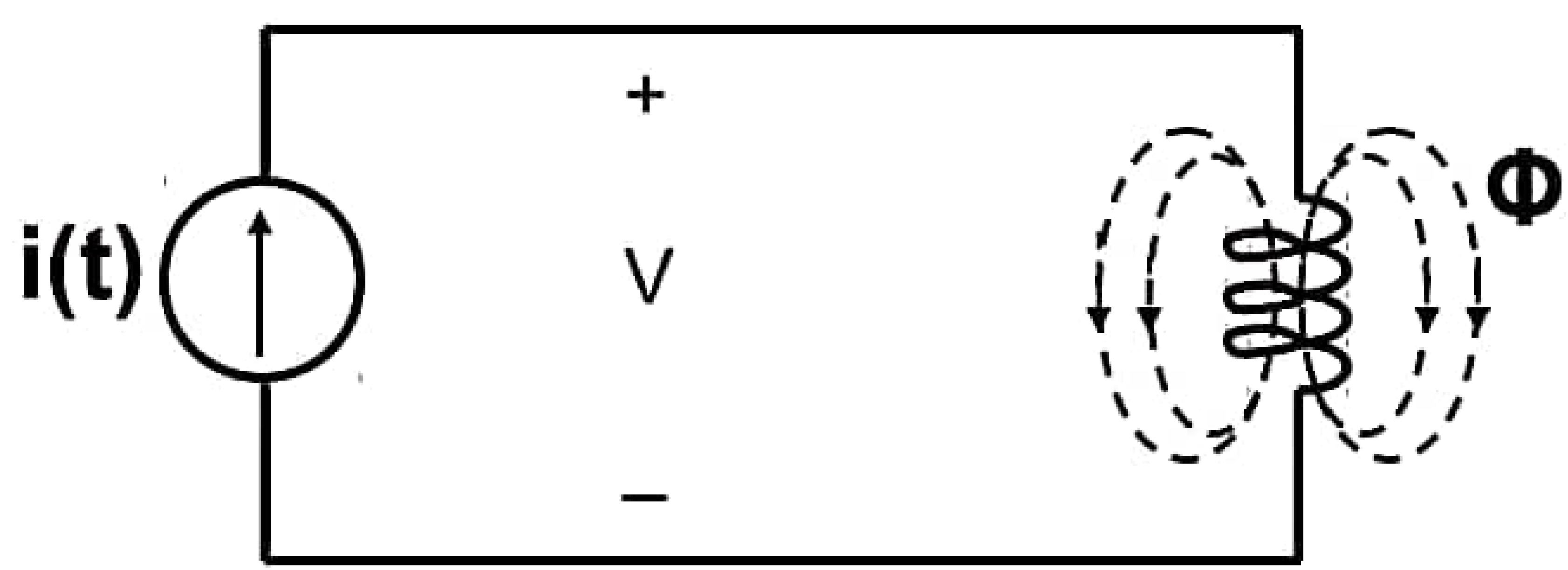


Fig. 1

- According to Faraday's Law, the voltage, *v* induced in the coil is proportional to N number of turns and rate of change of the magnetic flux, Φ;

$$v = N\frac{d\phi}{dt} \quad ....(1)$$

- But a change in the flux Φ is caused by a change in current, *i*.

Hence;

$$\frac{d\phi}{dt} = \frac{d\phi}{di}\frac{di}{dt} \quad ....(2)$$

Thus, (2) into (1) yields;

$$v = N \frac{d\phi}{di} \frac{di}{dt} \ldots\ldots.(3)$$

or

$$v = L \frac{di}{dt} \ldots\ldots.(4)$$

From equation (3) and (4) the self inductance L is define as;

$$L = N \frac{d\phi}{di} \quad [\mathrm{H}] \ldots\ldots.(5)$$

The unit is in Henrys (H)

b) Mutual Inductance

- When two inductors or coils are in close proximity to each other, magnetic flux caused by current in one coil links with the other coil, therefore producing the induced voltage.

- Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor.

Consider the following two cases:

- Case 1:

two coil with self – inductance L_1 and L_2 which are in close proximity which each other (Fig. 2). Coil 1 has N_1 turns, while coil 2 has N_2 turns.

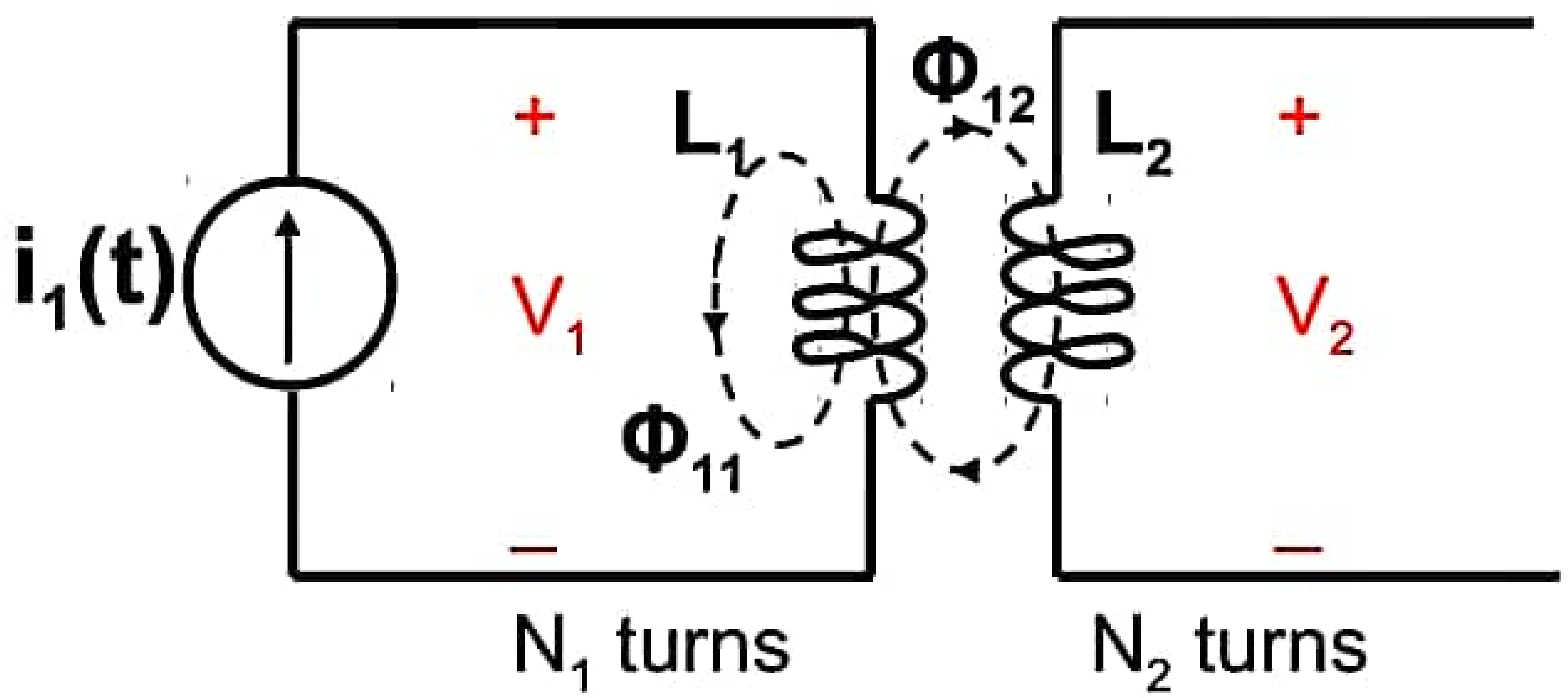


Fig. 2

- Magnetic flux Φ_1 from coil 1 has two components;

  * Φ_{11} links only coil 1.

  * Φ_{12} links both coils.

Hence; $\Phi_1 = \Phi_{11} + \Phi_{12}$ ……. (6)

Thus;

Voltage induces in coil 1

$$v_1 = N_1 \frac{d\phi_{11}}{di_1} \frac{di_1}{dt} = L_1 \frac{di_1}{dt} \text{......}(7)$$

Voltage induces in coil 2

$$v_2 = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21}\frac{di_1}{dt} \quad ....(8)$$

Subscript 21 in M_{21} means the mutual inductance on coil 2 due to coil 1

■ Case 2:

Same circuit but let current i_2 flow in coil 2.

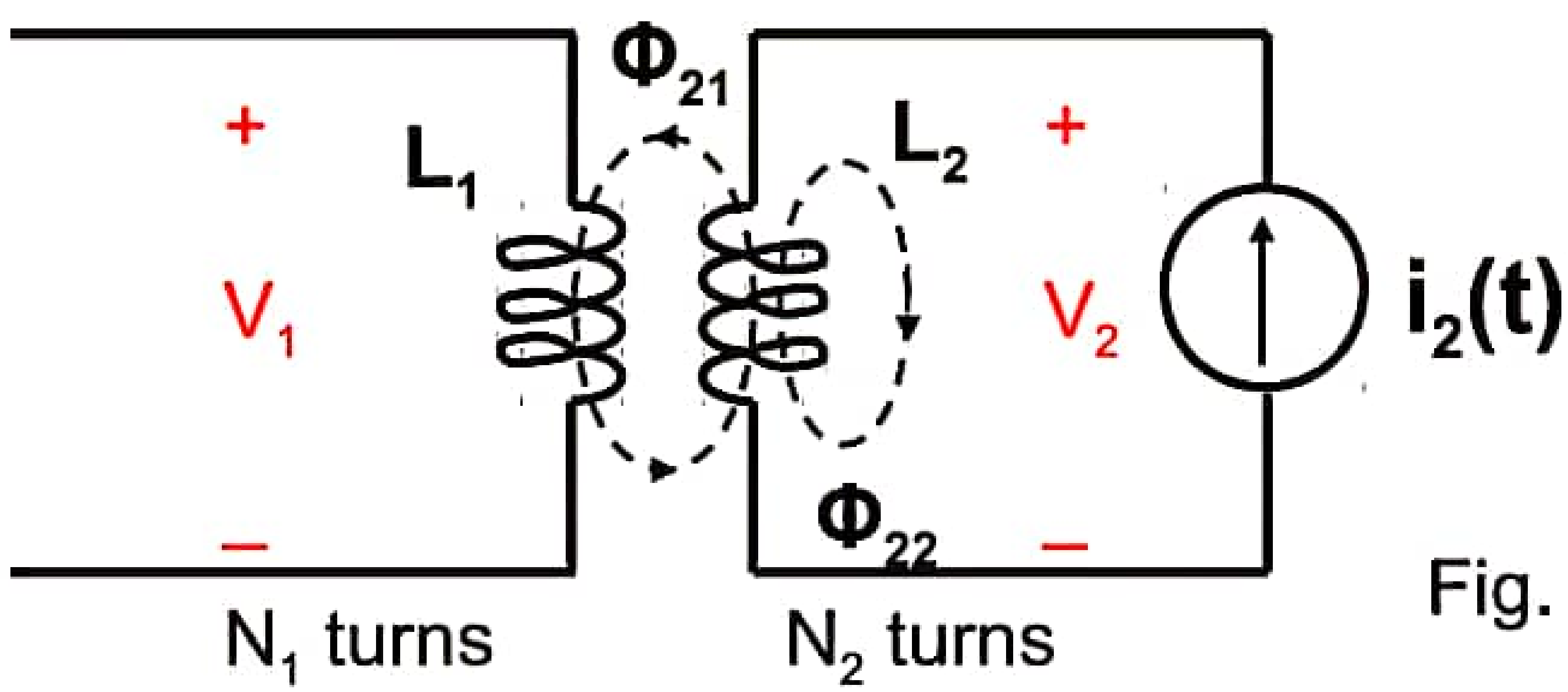


Fig. 3

■ The magnetic flux Φ_2 from coil 2 has two components:

* Φ_{22} links only coil 2.

* Φ_{21} links both coils.

Hence; $\Phi_2 = \Phi_{21} + \Phi_{22}$ ……. (9)

Thus;

Voltage induced in coil 2

$$v_2 = N_2 \frac{d\phi_{22}}{di_2}\frac{di_2}{dt} = L_2 \frac{di_2}{dt} \quad ....(10)$$

Voltage induced in coil 1

$$v_1 = N_1 \frac{d\phi_{21}}{di_2}\frac{di_2}{dt} = M_{12}\frac{di_2}{dt} \quad ....(11)$$

Subscript 12 in M_{12} means the Mutual Inductance on coil 1 due to coil 2

- Since the two circuits and two current are the same:

$$M_{21} = M_{12} = M$$

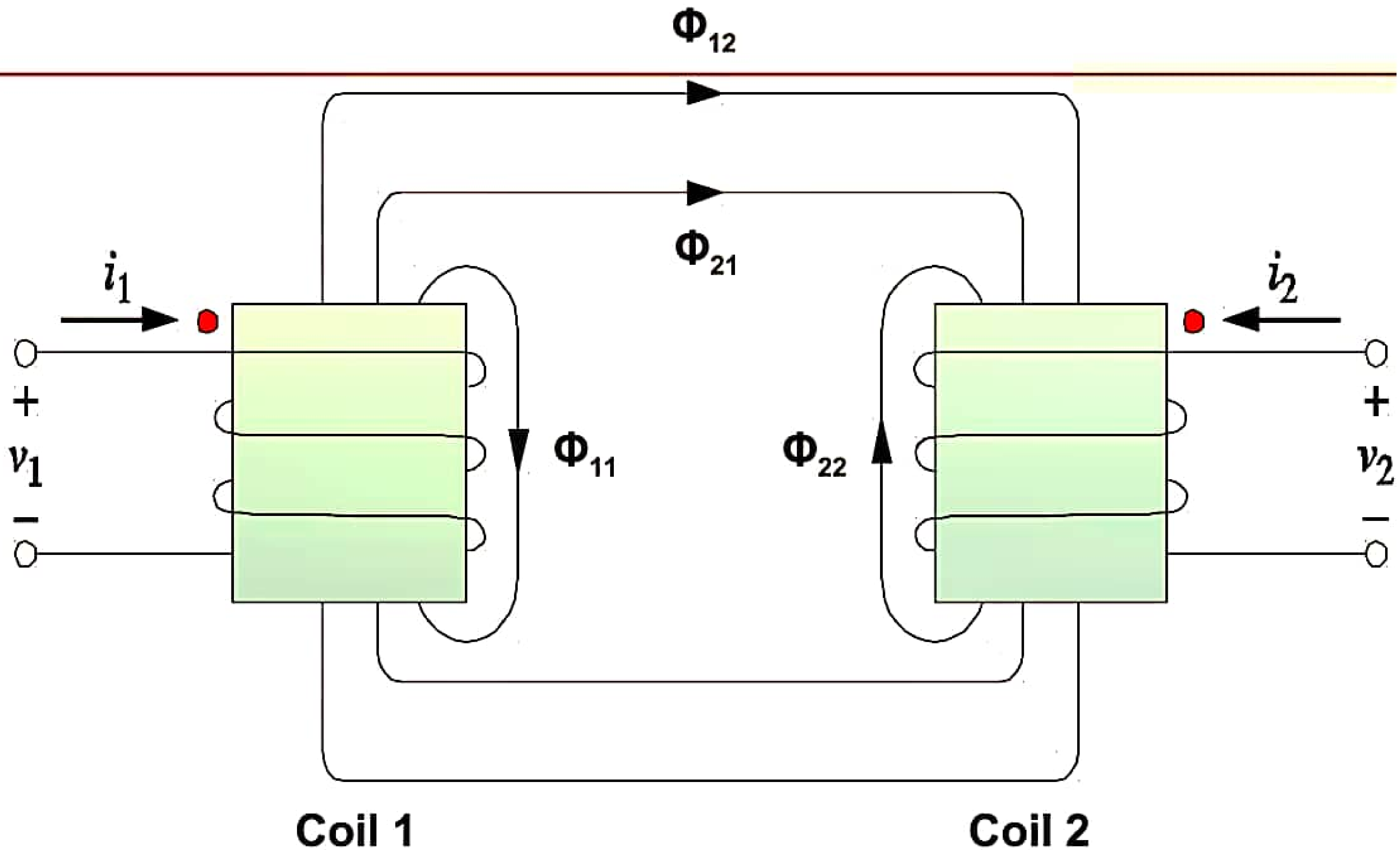
- Mutual inductance M is measured in Henrys (H)

COUPLING COEFFICIENT (k)

- It is measure of the magnetic coupling between two coils.
- Range of k : $0 \leq k \leq 1$
  - k = 0 means the two coils are NOT COUPLED.
  - k = 1 means the two coils are PERFECTLY COUPLED.
  - k < 0.5 means the two coils are LOOSELY COUPLED.
  - k > 0.5 means the two coils are TIGHTLY COUPLED.

DOT DETERMINATION

- Required to determine polarity of “mutual” induced voltage.
- A dot is placed in the circuit at one end of each of the two magnetically coupled coils to indicate the direction of the magnetic flux if current enters that dotted terminal of the coil.



Φ_{12}
Φ_{21}
i_1
i_2
+
v_1
−
+
v_2
−
Φ_{11}
Φ_{22}
Coil 1
Coil 2

- Dot convention is stated as follows:

  if a current ENTERS the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is POSITIVE at the dotted terminal of the second coil.

- Conversely, Dot convention may also be stated as follow:

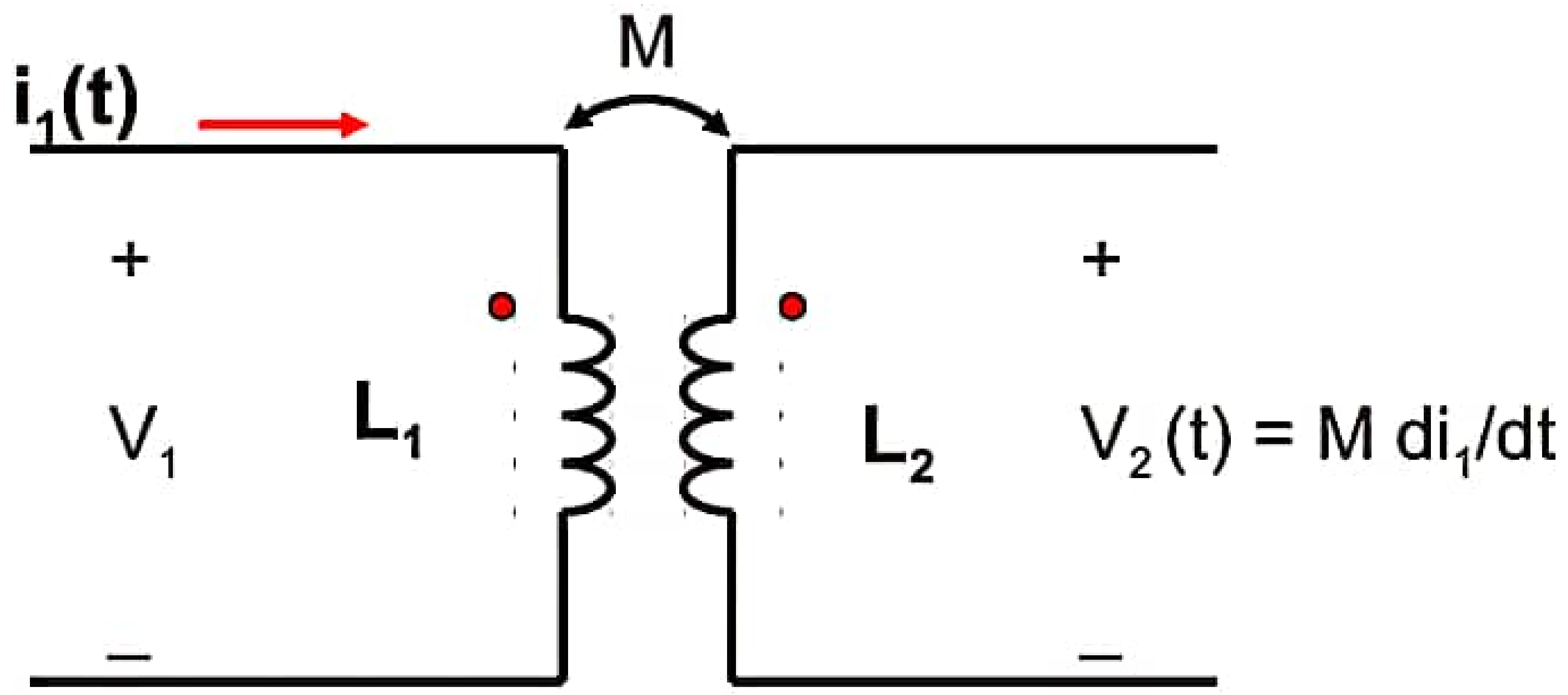
  if a current LEAVES the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is NEGATIVE at the dotted terminal of the second coil.

- The following dot rule may be used:

i. when the assumed currents both entered or both leaves a pair of couple coils by the dotted terminals, the signs on the L – terms.

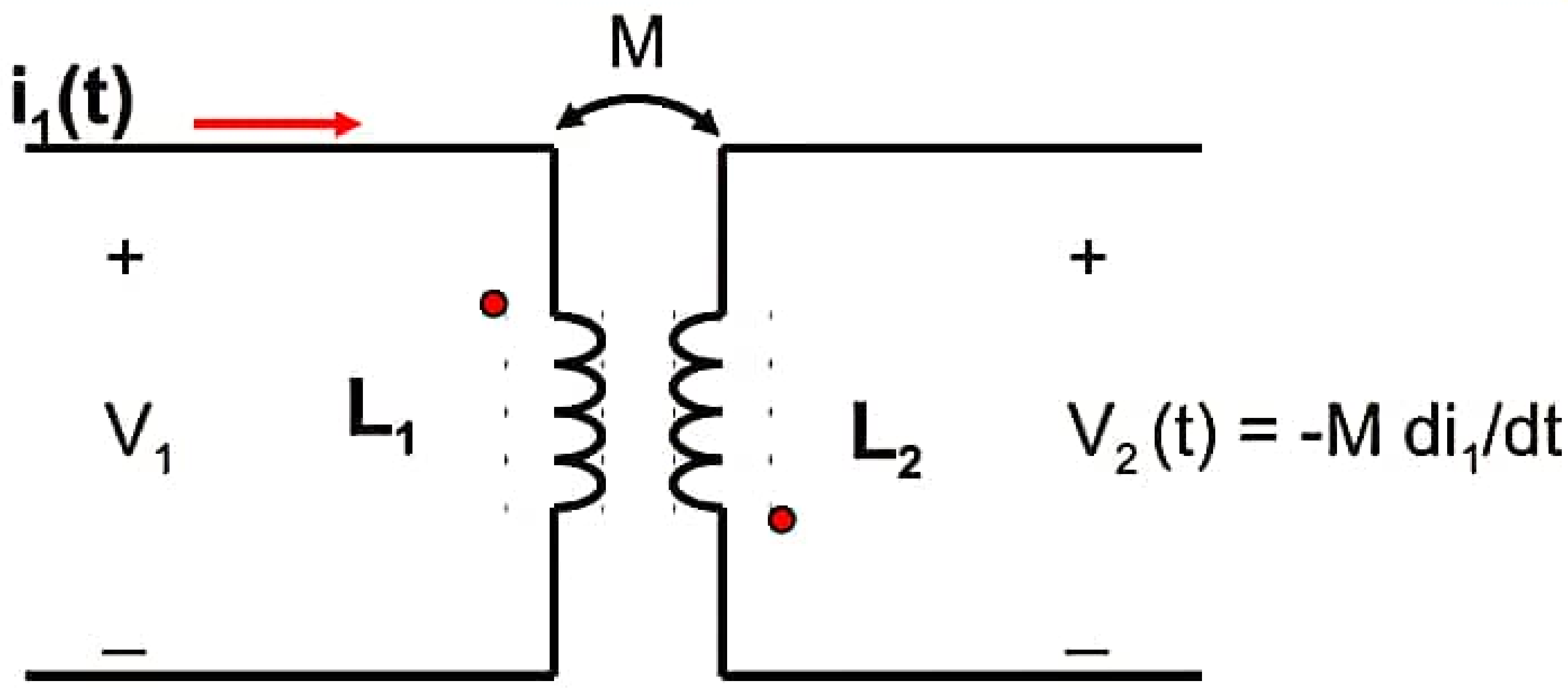
ii. if one current enters by a dotted terminals while the other leaves by a dotted terminal, the sign on the M – terms will be opposite to the signs on the L – terms.

- Once the polarity of the mutual voltage is already known, the circuit can be analyzed using *mesh method*.
- Application of the dot convention
- Example 1



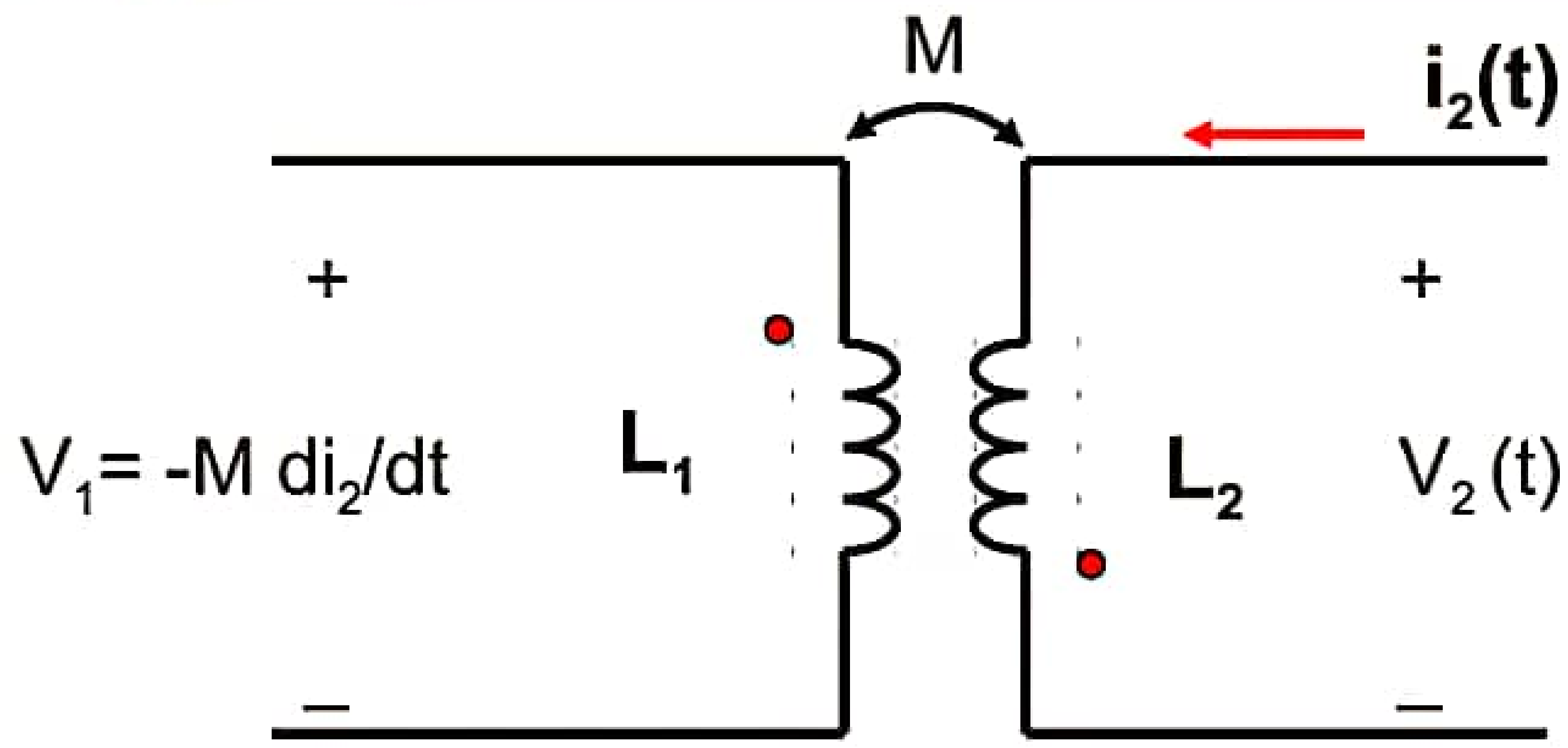
The sign of the mutual voltage v_2 is determined by the reference polarity for v_2 and the direction of i_1. Since i_1 enters the dotted terminal of coil 1 and v_2 is positive at the dotted terminal of coil 2, the mutual voltage is $M\,di_1/dt$

- Example 2

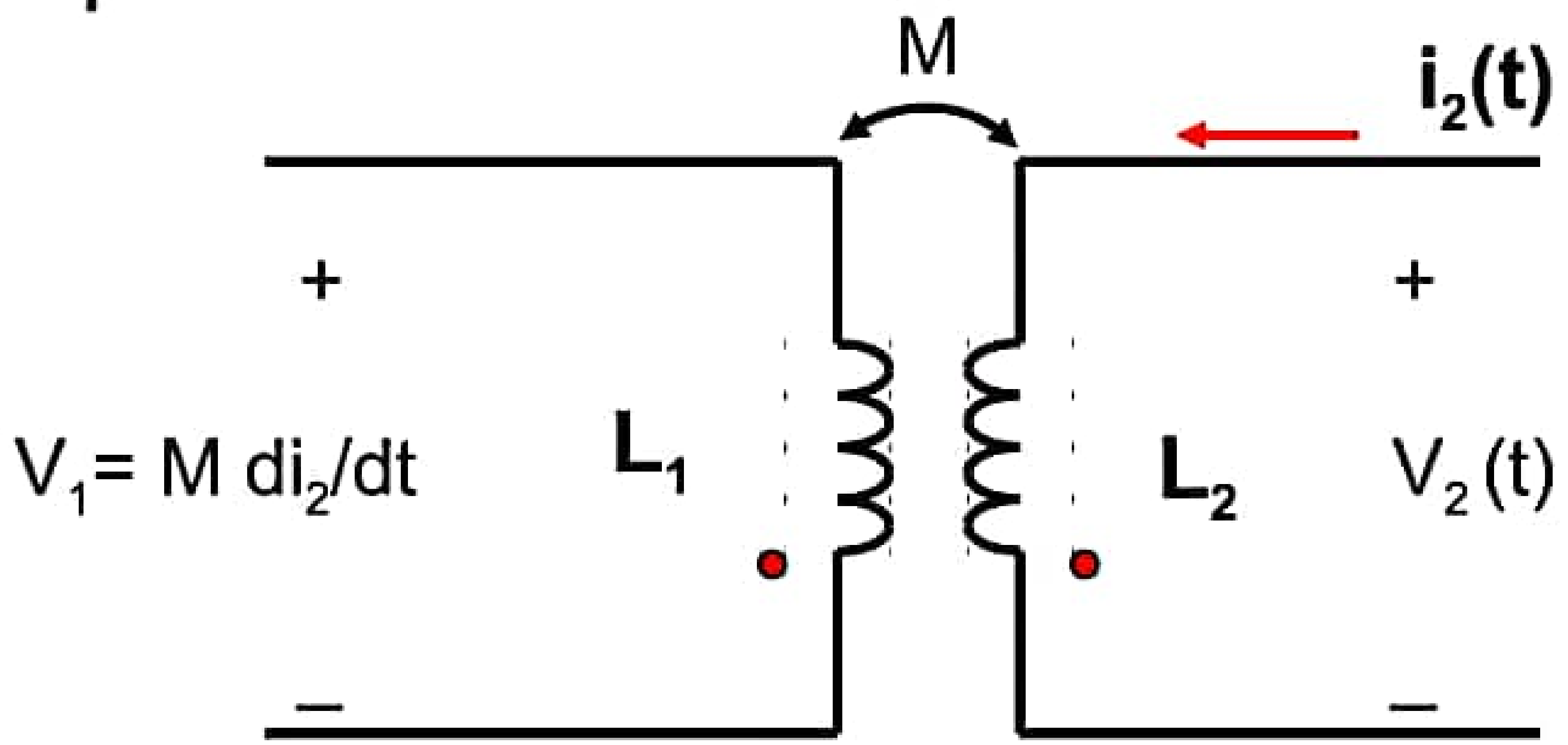


Current i_1 enters the dotted terminal of coil 1 and v_2 is negative at the dotted terminal of coil 2. the mutual voltage is $-M\, di_1/dt$

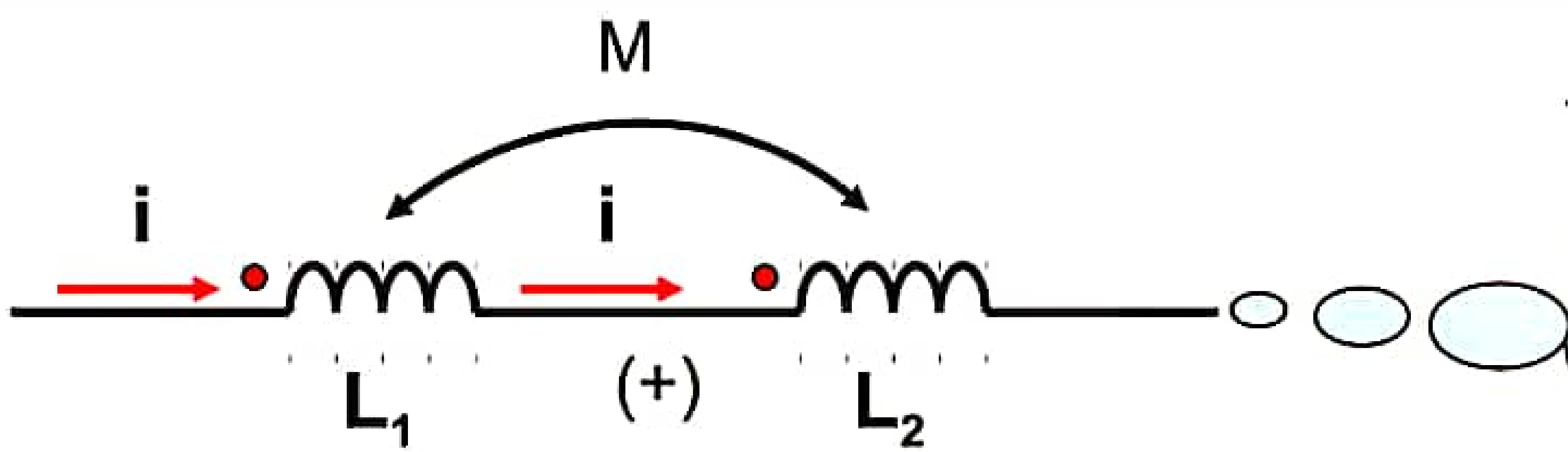
- Same reasoning applies to the coil in example 3 and example 4.
- Example 3

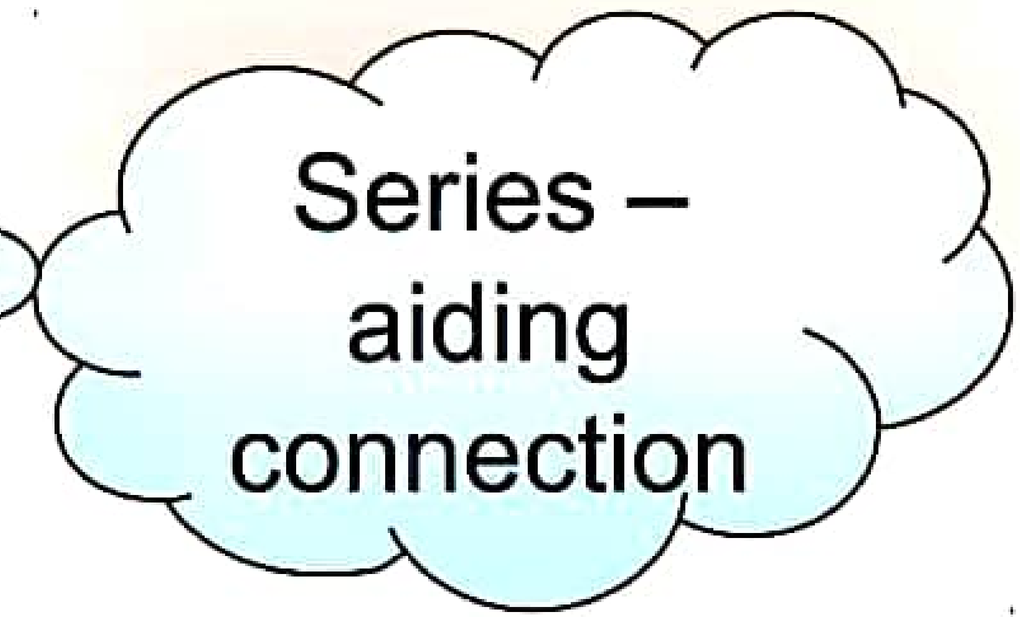


- Example 4

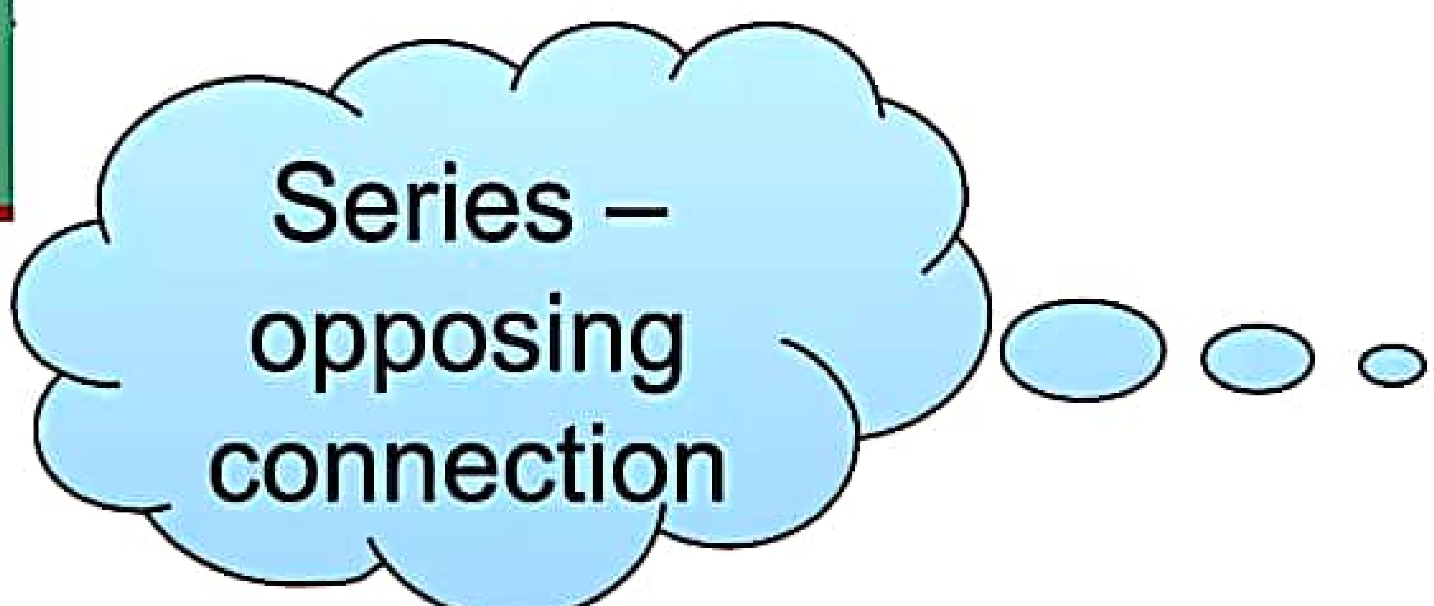


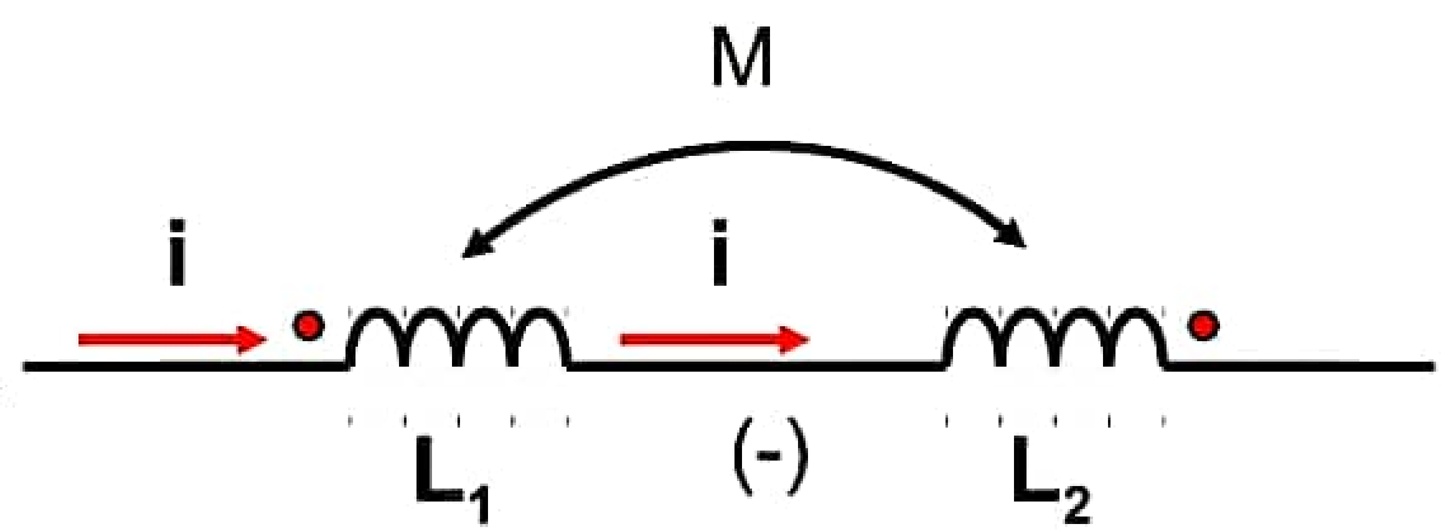
Dot convention for coils in series





$$L = L_1 + L_2 + 2M$$

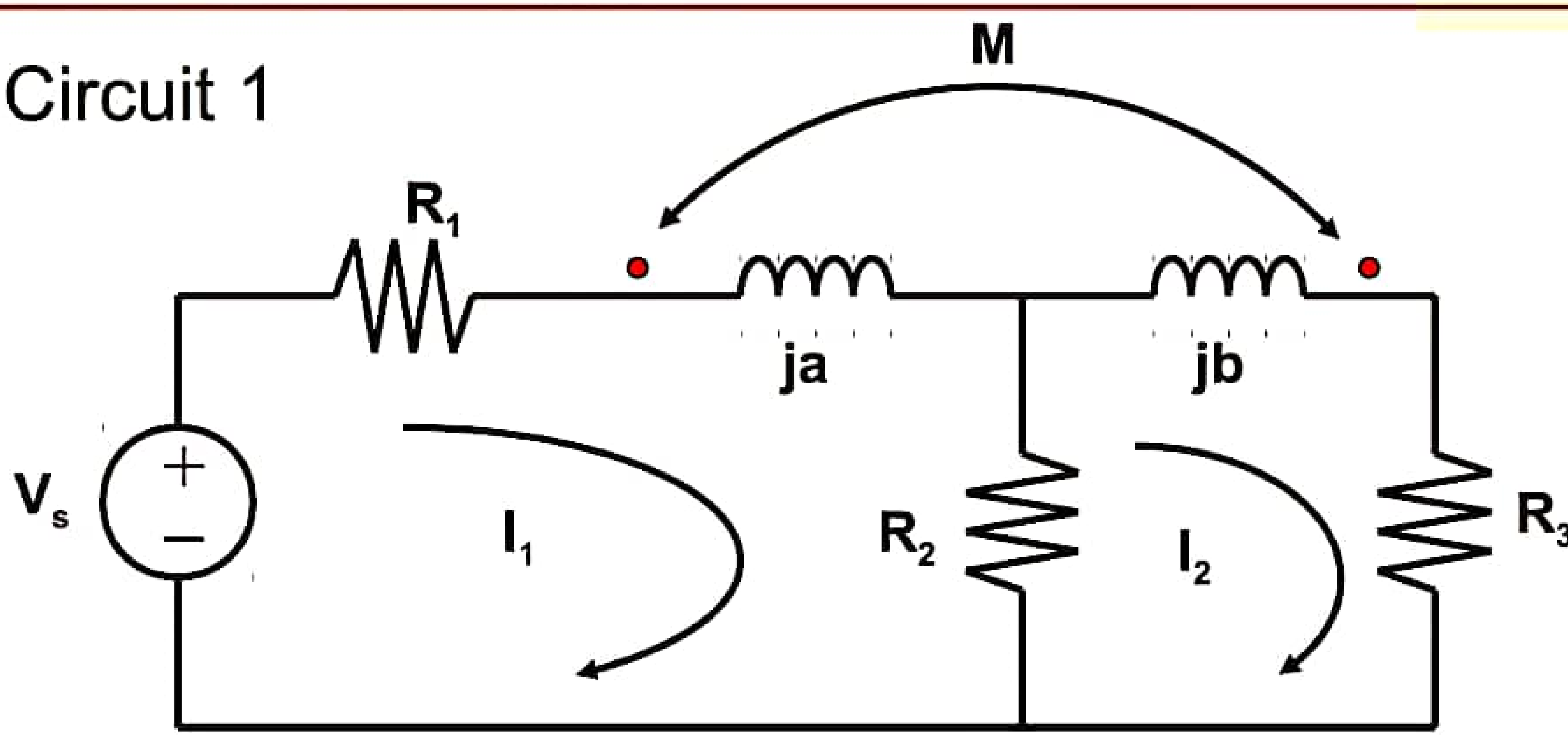




$$L = L_1 + L_2 - 2M$$

Below are examples of the sets of equations derived from basic configurations involving mutual inductance

- Circuit 1

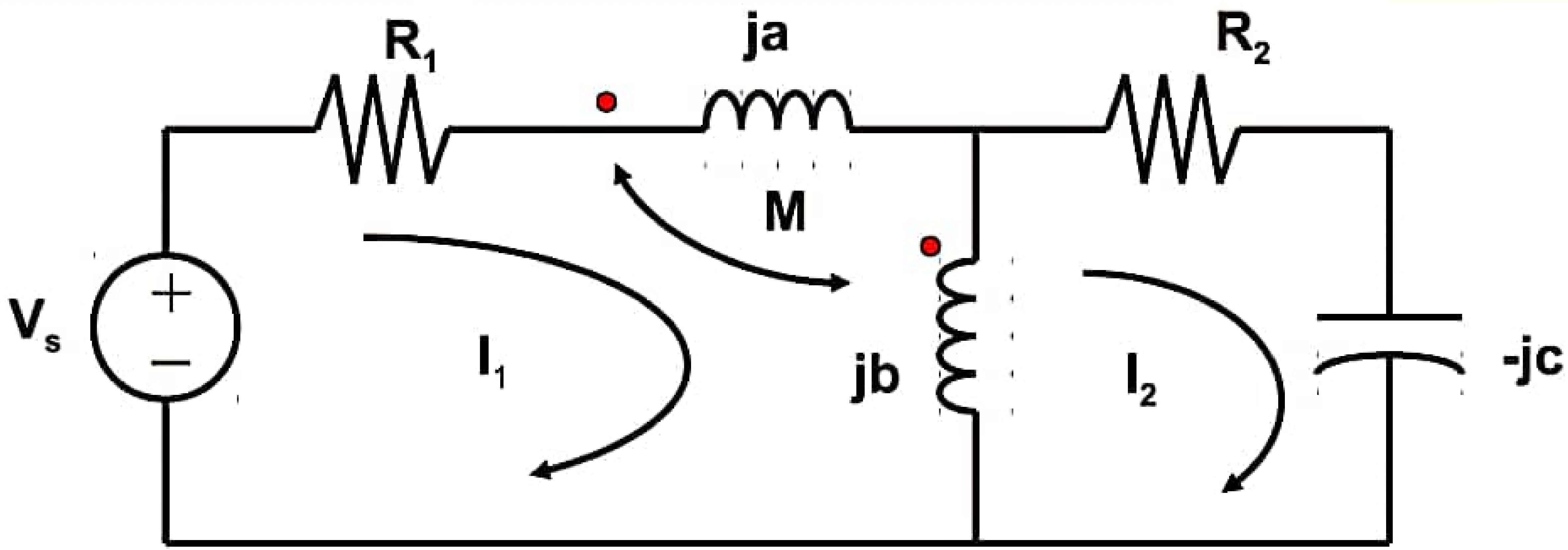


Solution:

$$\text{KVL } \mathrm{I}_1 : (R_1 + R_2 + ja)I_1 - MI_2 = V_s \ldots\ldots(1)$$

$$\text{KVL } \mathrm{I}_2 : -R_2 I_1 + (R_2 + R_3 + jb)I_2 - MI_1 = 0 \ldots\ldots(2)$$

■ Circuit 2

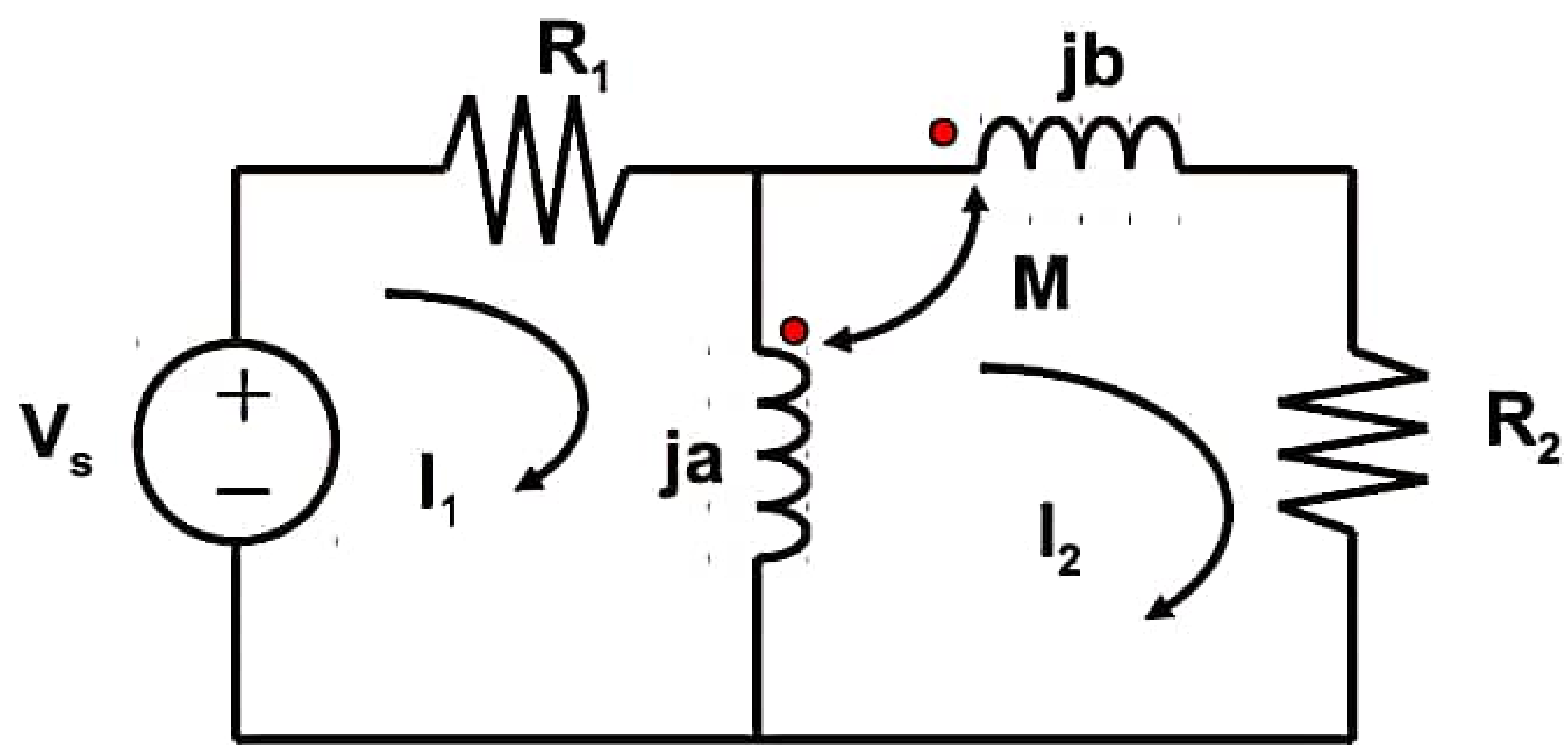


Solution:

$$\text{KVL I}_1 : (R_1 + ja + jb)I_1 - jbI_2 + M(I_1 - I_2) + MI_1 = V_s ....(1)$$

$$\text{KVL I}_2 : -jbI_1 + (R_2 + jb - jc)I_2 - MI_1 = 0 ......(2)$$

Circuit 3

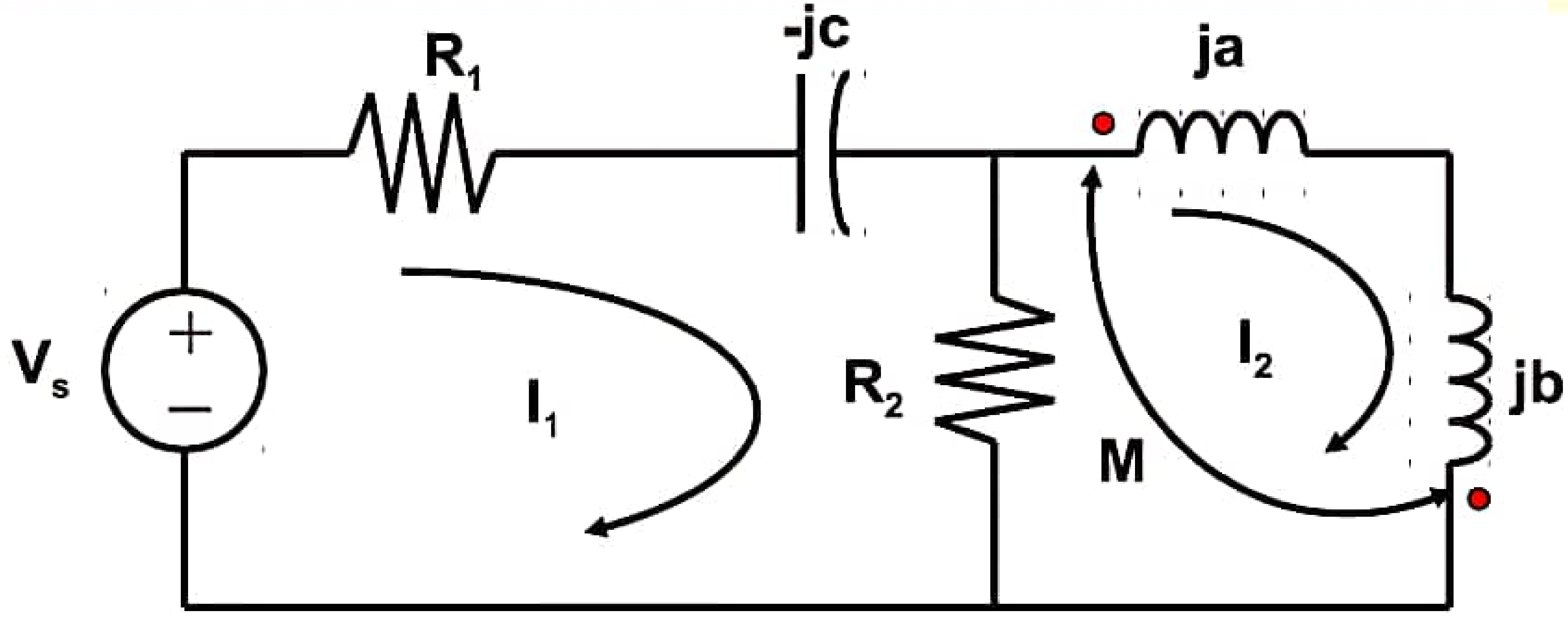


Solution:

$$\text{KVL } I_1 : (R_1 + ja)I_1 - jaI_2 + MI_2 = V_s \text{.......}(1)$$

$$\text{KVL } I_2 : -jaI_1 + (R_2 + ja + jb)I_2 - MI_2 - M(I_2 - I_1) = 0 \text{.......}(2)$$

Circuit 4

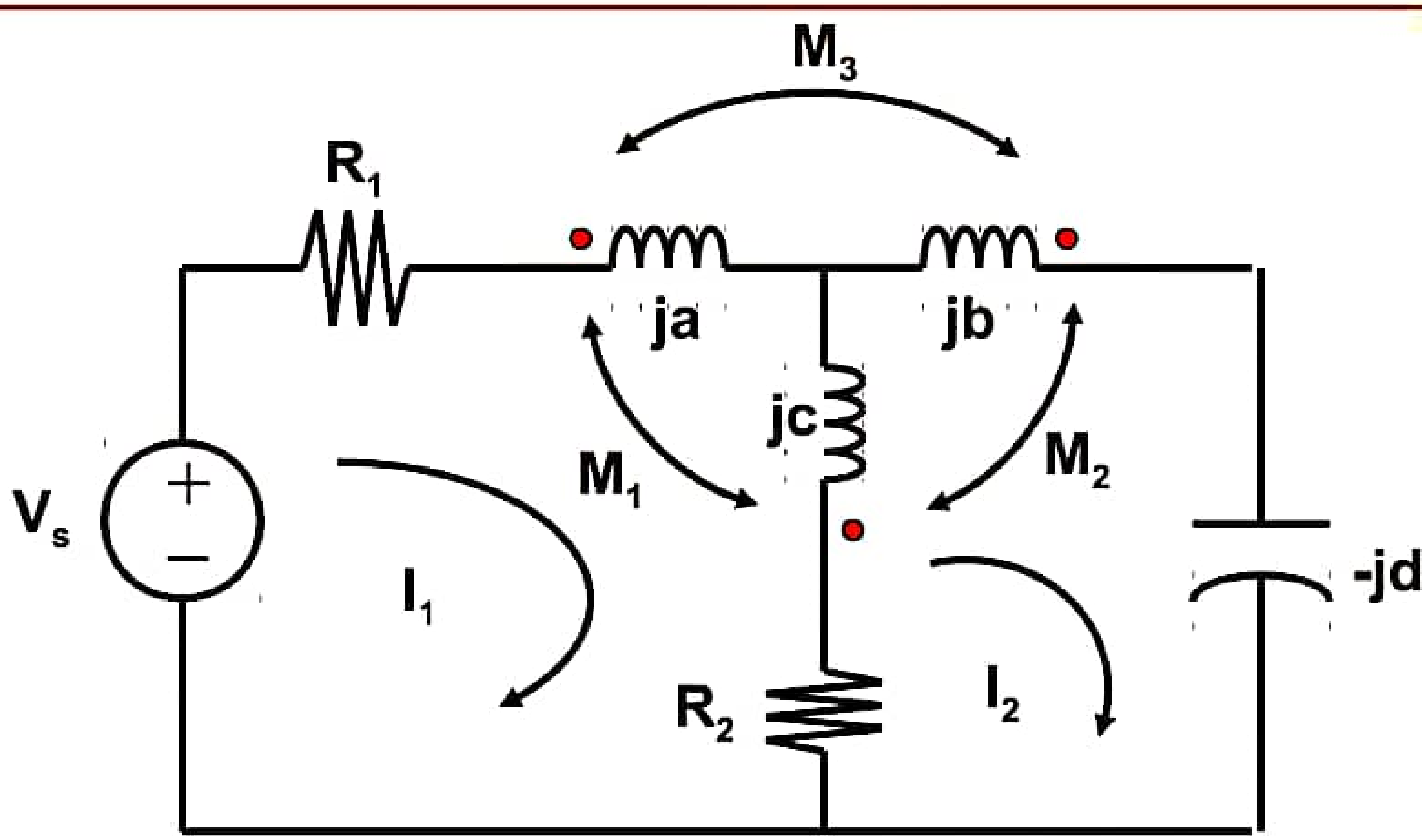


Solution:

$$\text{KVL I}_1 : (R_1 + R_2 - jc)I_1 - R_2 I_2 = V_s \text{.......}(1)$$

$$\text{KVL I}_2 : -R_2 I_1 + (R_2 + ja + jb)I_2 - 2MI_2 = 0 \text{......}(2)$$

Circuit 5



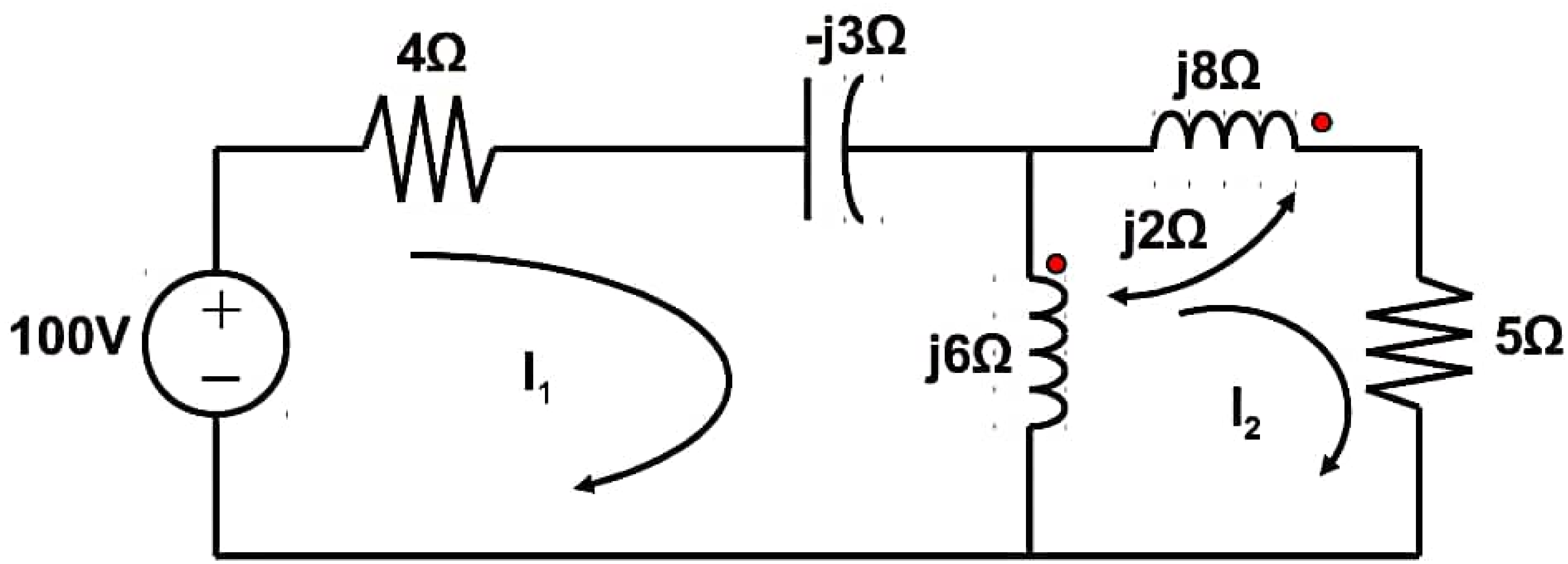
Solution:

$$\text{KVL } I_1 : (R_1 + R_2 + ja + jc)I_1 - (R_2 + jc)I_2 - M_3 I_2 - M_1(I_1 - I_2) - M_1 I_1 + M_2 I_2 = V_s \text{.......}(1)$$

$$\text{KVL } I_2 : -(R_2 + jc)I_1 + (R_2 + jc + jb - jd)I_2 + M_1 I_1 - M_2 I_2 - M_3 I_1 - M_2(I_2 - I_1) = 0 \text{.......}(2)$$

Example 1

Calculate the mesh currents in the circuit shown below



Solution

$$\text{KVL } \mathrm{I}_1 : (4 + j3)I_1 - j6I_2 - j2I_2 = 100$$

$$(4 + j3)I_1 - j8I_2 = 100 \ldots\ldots(1)$$

$$\text{KVL } \mathrm{I}_2 : -j6I_1 + (5 + j14)I_2 + j2I_2 + j2(I_2 - I_1) = 0$$

$$-j8I_1 + (5 + j18)I_2 = 0 \ldots\ldots(2)$$

In matrix form;

$$\begin{bmatrix} 4 + j3 & -j8 \\ -j8 & 5 + j18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

The determinants are:

$$\Delta = \begin{vmatrix} 4+j3 & -j8 \\ -j8 & 5+j18 \end{vmatrix} = 30 + j87$$

$$\Delta_1 = \begin{vmatrix} 100 & -j8 \\ 0 & 5+j18 \end{vmatrix} = 500 + j1800$$

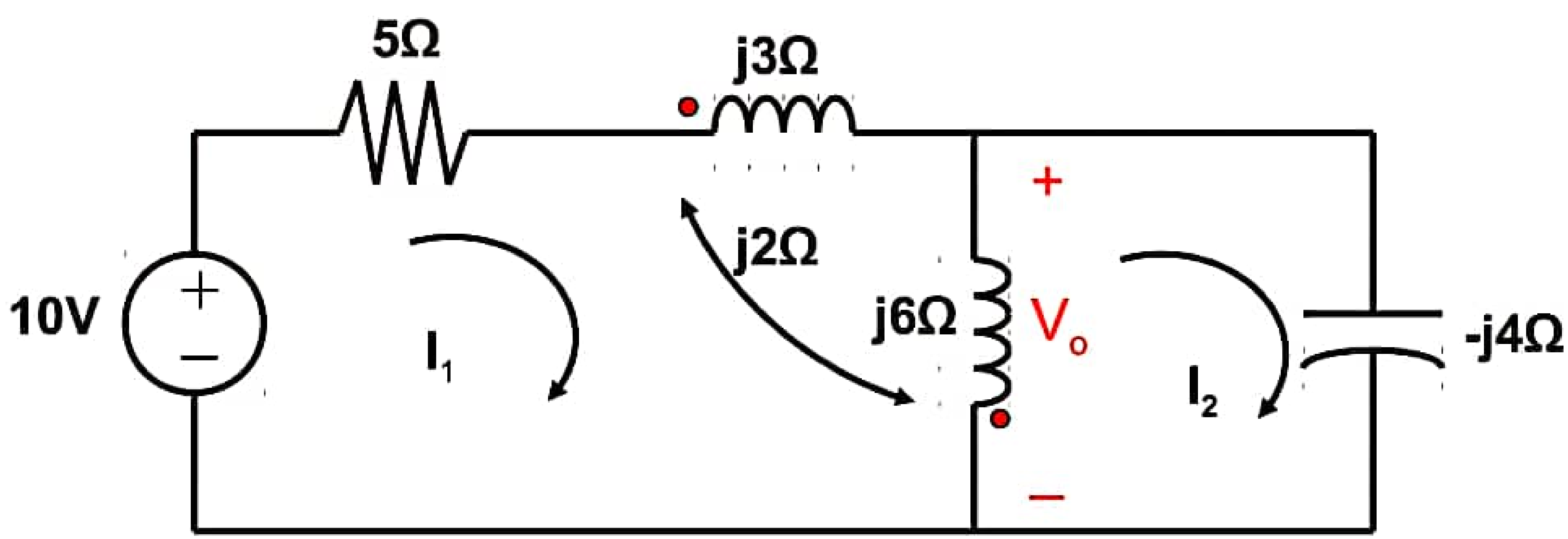
$$\Delta_2 = \begin{vmatrix} 4+j3 & 100 \\ -j8 & 0 \end{vmatrix} = j800$$

Hence: $\therefore I_1 = \dfrac{\Delta_1}{\Delta} = 20.3\angle 3.5^\circ A$

$\therefore I_2 = \dfrac{\Delta_2}{\Delta} = 8.7\angle 19^\circ A$

Example 2

Determine the voltage V_0 in the circuit shown below.



Solution

$$\text{KVL } \mathrm{I}_1 : (5+j9)I_1 - j6I_2 - j2(I_1 - I_2) - j2I_1 = 10$$

$$(5+j5)I_1 - j4I_2 = 10 ....(1)$$

$$\text{KVL } \mathrm{I}_2 : -j6I_1 + j2I_2 + j2I_1 = 0$$

$$-j4I_1 + j2I_2 = 0 ....(2)$$

In matrix form;

$$\begin{bmatrix} 5+j5 & -j4 \\ -j4 & j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

Answer:

$$I_1 = 1.47 + j0.88$$

$$I_2 = 2.94 + j1.76$$

$$V_0 = j6(I_1 - I_2) - j2I_1 \text{ or}$$

$$V_o = -[j6(I_2 - I_1) + j2I_1] \text{ or}$$

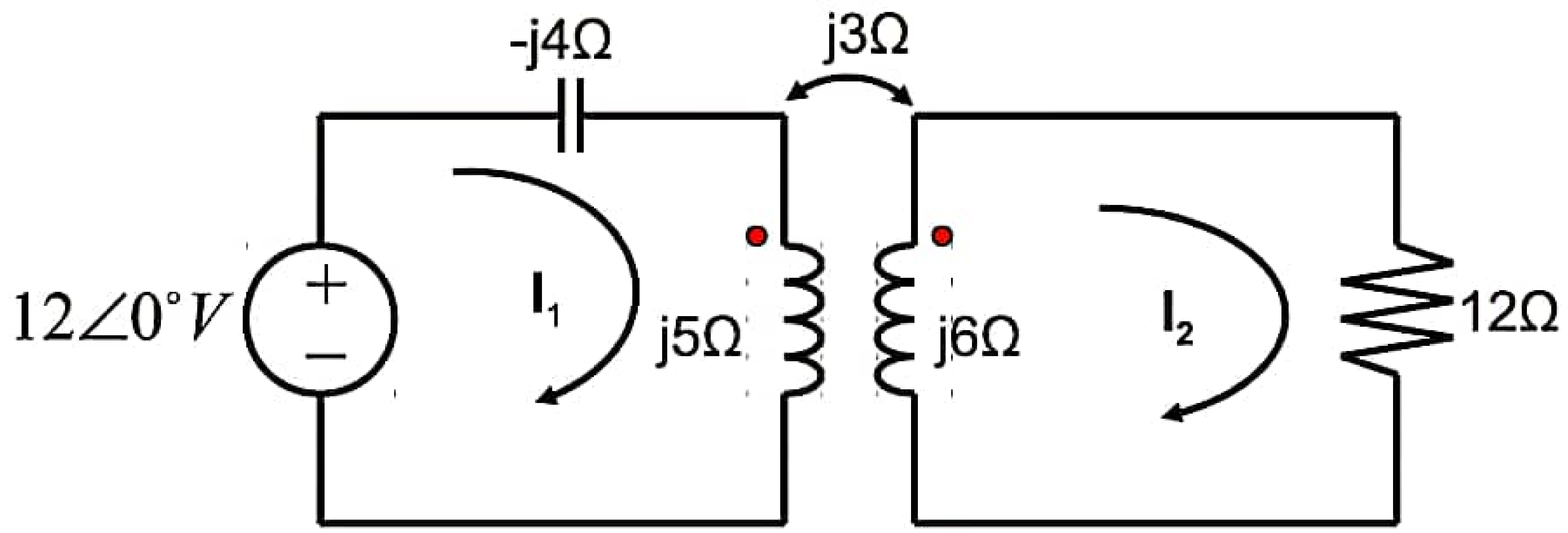
$$V_o = -j4I_2$$

hence,

$$V_o = 7.04 - j11.76$$

Example 3

Calculate the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit below.



Solution

For coil 1, KVL gives

$$-12 + (-j4+j5)I_1 - j3I_2 = 0$$

Or

$$jI_1 - j3I_2 = 12 \quad \text{—— (1)}$$

For coil 2, KVL gives

$$-j3I_1 + (12 + j6)I_2 = 0$$

Or

$$I_1 = \frac{(12 + j6)I_2}{j3} = (2 - j4)I_2 \quad \text{—— (2)}$$

Substituting (2) into (1) :

$$(j2 + 4 - j3)I_2 = (4 - j)I_2 = 12$$

Or

$$I_2 = \frac{12}{4 - j} = 2.91\angle 14.04^\circ \text{ A} \quad \text{—— (3)}$$

From eqn. (2) and (3),

$$I_1 = (2 - j4)I_2 = (4.472\angle -63.43^\circ)(2.91\angle 14.04^\circ)$$

$$= 13.01\angle -49.39^\circ \text{ A}$$